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Appendix to "Superstars and Mediocrities:" Generalization to Gradual Learning

This note extends the model in "Superstars and Mediocrities"[29] by allowing information about talent to be revealed over time. While the optimal solution is analogous to that of the basic model, the case of credit constrained individuals is altered by the opportunity to save. I will show that, instead of mitigating the inefficiency caused by a credit constraint, saving will actually make things worse. It lowers exit rates even further below optimal because some veterans of below-average talent will stay in the industry.

In the basic model, individuals have essentially two-period careers, with the relative length of the second "veteran" period described by T. All uncertainty about an individual's talent is resolved at a single point in time, so the only variable of choice is the exit threshold at that point. When new information about talent arrives at several points in time, then the decision to continue must take into account the option of exiting at a later time. Without further constraints, this would be a standard optimal stopping problem, introduced into the theory of labor markets by Jovanovic (1979). In this appendix I explore the general implications of a similar problem, but when jobs are scarce, learning is public and industry-specific, and workers have finite careers and cannot commit to long-term contracts.

Assumptions

1. Each firm employs one worker per period whose output is $y_t = \theta + \varepsilon_t$, where θ is the worker's talent and ε_t an i.i.d. error term.

2. There is an unlimited supply of individuals willing to work at outside wage w_0.

3. Individual careers last up to $1 + T$ periods.

4. The cumulative distribution functions of θ and ε are strictly increasing, continuous, and yield finite moments.

5. $\{\hat{\theta}_t, t\}$ is a sufficient statistic for $\hat{\theta}_{t+1}$, where $\hat{\theta}_t \equiv E[\theta | y_1, \ldots, y_t]$ is the expected level of talent at tenure t (i.e. after t periods of work).

6. Firms are infinitely lived and maximize average expected per-period profits.

The expectation $\hat{\theta}_t$ is taken with respect to the known distributions f_θ and f_ε. For the novices, no output has yet been observed, so $\hat{\theta}_0 = \bar{\theta}$ for all of them. Since predictions are unbiased by definition, $\mathrm{E}[\hat{\theta}_{t+s} | \hat{\theta}_t] = \hat{\theta}_t$ for any $s = 1, \ldots, T - t$. Period t perceived talent $\hat{\theta}_t$ will often be simply referred to as talent. A crucial implication of the assumptions is that the distribution of prediction errors does not become degenerate in finite time: there is always some chance that the individual is better than he is expected to be.

Going forward in time, the estimate of any particular worker's talent becomes more

[29] "Superstars and Mediocrities: Market Failure in The Discovery of Talent" (2008). Marko Terviö. University of California, Berkeley.

precise; it gets closer to the true value in expectation and moves about less. In terms of a whole cohort, the distribution $f_{\hat{\theta}_t}$ starts as a degenerate distribution at $\bar{\theta}$, and then becomes more spread out. Without filtering, it would become more like the true distribution f_θ; with filtering, more of the lower types, as well as some unlucky higher types, get discarded as time goes by.

Other assumptions

7. There is a unit measure of firms.
8. Price of output is fixed at unity.

These assumptions are made in order to simplify the notation. Also, in this section it will be assumed that constrained individuals cannot accept to consume less than the outside wage.[30] Extending the analysis to allow for free entry, an endogenous output price, and a positive payment by novices, is straightforward in light of Section 3.

Social Planner's Problem

The variable of choice is a stopping (exit) policy $\boldsymbol{\psi} = \{\psi_1, \ldots, \psi_T\}$, which consists of T separate exit thresholds. Analogously to the single exit threshold of the basic model, this policy states that an individual with talent $\hat{\theta}_t < \psi_t$ will exit the industry. Since everyone looks identical at $t = 0$, there is no meaningful choice for ψ_0 (besides $\bar{\theta}$). On the other hand, after $T + 1$ periods, the individual will retire anyway, and the updating of $\hat{\theta}$ based on y_{T+1} is useless. Hence there are in total T points in time where a decision to continue or stop has to be made. A decision to exit is final, because after the exit no new information will ever arrive that could change the decision.

The average level of talent in the industry depends on the whole stopping policy. In line with earlier notation, denote the maximal solution by $A^* \equiv \max_{\boldsymbol{\psi}} A(\boldsymbol{\psi})$. This would be the object of a surplus-maximizing social planner, as well as of a firm who could keep individuals for the whole lifetime at a fixed wage. The optimal solution must adhere to the following variant of the fixed point result in Proposition 1.

Proposition 8 *In the optimal solution, $\psi_T = A^*$.*

The proof is omitted as it is basically the same as in the basic model. Intuitively, given an individual with just one period left, the optimal decision of whether to retain him or not depends solely on his expected talent for his final period; there is no value for any further information about him. Thus he should be retained if and only if he contributes positively to the average talent in the industry.[31] At the social optimum, this means that he should be

[30] In terms of the basic model, now $I = 1$, $P = 1$, and $b = 0$.

[31] Again, with a positive discount rate the optimal exit threshold would be below the optimal average, which in turn is decreasing in the discount rate (since hiring novices is an investment). Discount factors would complicate the notation without affecting the comparison between different cases.

retained if and only if $\hat{\theta}_T \geq A^*$.

Market Equilibrium

As in the basic model, equilibrium wages are determined on the spot market, taking into account that individuals on their last period before retirement have a trivial exit decision—for them it's all about the current wage.

Proposition 9 *Wages are* $w(\hat{\theta}) = \hat{\theta} - \psi_T + w_0$.

The proof is a combination of two observations. First, for individuals at tenure T, the value of continuing in the industry is simply $w(\hat{\theta}) - w_0$. With a continuum of types, the lowest type to stay is one who gets exactly the outside wage w_0 by doing so, hence $w(\psi_T) = w_0$ regardless of the value of ψ_T. Second, firms must be indifferent between hiring a type ψ_T and any worker in the industry. The difference in current period wage between any two workers must therefore be equal to the difference in their expected talent.□

In Jovanovic (1979, p. 976), workers have infinite lives, and this "*assumption justifies the exclusion of age as an explicit argument from the wage function.*" Here that exclusion follows from the existence of a spot market for talent, and from the market price of talent being constant in steady state.

Proposition 10 *Given any* $\psi_T \geq \bar{\theta}$*, the optimal exit policy for risk-neutral individuals is strictly increasing in tenure:* $\psi_t < \psi_{t+1}$.

Proof by backwards induction. First, consider an individual of type $\hat{\theta}_T = \hat{\theta}$ at tenure T. His payoff or "value function" is

$$V_T(\hat{\theta}) = \max\{0, \hat{\theta} - \psi_T\}. \tag{37}$$

The value function gives the excess expected utility from continuing as opposed to exiting, and zero if that difference is negative.

Next consider an individual of type $\hat{\theta}_{T-1} = \hat{\theta}$ at tenure $T-1$. If he decides to continue, he gets lifetime expected utility

$$\tilde{V}_{T-1}(\hat{\theta}) = \hat{\theta} - \psi_T + \mathrm{E}[V_T(a)|\{\hat{\theta}, T-1\}]. \tag{38}$$

The expected utility is taken with respect to $f_{\hat{\theta}_T|\hat{\theta}_{T-1}}(a|\hat{\theta})$. Since the expectation is increasing in the prior, also (38) is strictly increasing in $\hat{\theta}$. Since the distribution functions were assumed to be continuous, this is also continuous in $\hat{\theta}$. The optimal exit threshold ψ_{T-1} is defined by $\tilde{V}_{T-1}(\psi_{T-1}) = 0$.

To see that $\psi_{T-1} < \psi_T$, notice that $\tilde{V}_{T-1}(\psi_T) > 0$, because the expectation is strictly positive at $\hat{\theta} = \psi_T$ (recall that the distribution of prediction errors does not become degenerate in finite time). Denote by V (without tilde) the value function that incorporates the current period optimal exit policy:

$$V_{T-1}(\hat{\theta}) = \max\{0, \hat{\theta} - \psi_T + \mathrm{E}[V_T(a)|\{\hat{\theta}, T-1\}]\}. \tag{39}$$

This is zero for $\hat{\theta} \leq \psi_{T-1}$, and strictly increasing for $\hat{\theta} > \psi_{T-1}$.

Completing the induction backwards in time is straightforward. The value function V_t is always zero below ψ_t, where there is a kink, and then has positive slope above. Hence $\tilde{V}_{t-1}(\psi_t) > 0$ and $\psi_{t-1} < \psi_t$.□

Intuitively, of two workers of the same expected ability, the younger one has always more upside potential because the prediction about his talent is less precise.[32] The standards for hiring should therefore be tougher for older workers. In terms of the market equilibrium, the willingness to pay for a job slot is higher for a younger individual: paying for continuation today includes the option to continue tomorrow, and other things equal, an option on an asset with higher variance is more valuable.

Unconstrained Individuals

If individuals are risk neutral and not credit constrained, then market equilibrium must be efficient so that $\psi_T = A^*$. Again, the inability to commit to long-term contracts is inconsequential when individuals can pay their first employer for the expected value of future rents which are made possible by that initial job opportunity. Competition from novices forces incumbent workers to follow the socially optimal exit policy. This policy is illustrated in Figure 1 as the smoothly increasing graph from $\{0, \bar{\theta}\}$ to $\{T, A^*\}$.[33] All possible individual paths for $\hat{\theta}$ must start at $\bar{\theta}$; an individual stays in the industry until retirement if and only if the path stays above the optimal exit policy throughout. At each point in time, the wages are described by the vertical difference with the horizontal line at A^*, on which they are equal to the outside wage.

[Figure 1: Efficient Benchmark.]

With many potential points in career for exiting, the breakdown of the workforce by tenure can no longer be captured by the fraction of novices. As in the basic model, more novices are hired in the efficient case, but the exit rates (hazard rates of exit) are in general difficult to solve.

[32] There is also more lifetime left for that talent to be used, but the option value logic applies even with infinite lifetimes.

[33] The figures are drawn for a large number of periods T so that time looks continuous.

Constrained Individuals

Proposition 11 *If individuals are credit constrained, then $\psi_T = \bar{\theta}$, and no one will exit while $\hat{\theta}_t > \bar{\theta}$.*

Proof. Novices are not scarce, so they cannot get more than the outside wage w_0. By assumption of being constrained, they cannot get less either. Therefore $w(\bar{\theta}) = w_0$. But then anyone with expected talent above the population mean is making rents and does not exit. Also, by Proposition 9, w_0 must be the wage of last period's threshold type $\hat{\theta}_T = \psi_T$. Therefore $\psi_T = \bar{\theta}$.□

In contrast to the basic model, here the definition of mediocrity is age-dependent. A mediocre individual is above the population mean, but below the optimal exit threshold for his tenure. As in the basic model, there is too little exit when individuals are credit-constrained. Mediocre individuals take up job slots that would be in better use with novices. The mediocrities are illustrated by the light shaded region in Figure 2. The wage is now equal to w_0 at the horizontal line, and the rents at any point in time are described by the vertical distance from it.

If constrained individuals require at least the outside wage w_0 regardless of past earnings, then the actual exit policy is $\psi_t = \bar{\theta}$ at all t. This behavior would only arise if there were no saving at all, e.g. if individuals were infinitely risk averse or impatient.

The Phenomenon of Has-beens It seems reasonable to assume that individuals could save at least some of their rents. In this case the actual exit decision becomes path-dependent. Novices still cannot pay for jobs because they have had no opportunity to accumulate savings; this pins down wages and the tenure-T exit threshold. However, now some below-mean individuals have enough savings to be able to buy a job and continue in the industry. Whether they want to depends on how far below the mean they are and how much savings they have accumulated.

Proposition 12 *If individuals are credit constrained, risk neutral, and it is possible to save, then some veterans of tenure $t = 2, \ldots, T-1$ will not exit even if $\hat{\theta}_t < \bar{\theta}$.*

Proof. We know that $w\left(\bar{\theta}\right) = w_0$ and $\psi_T = \bar{\theta}$ by Propositions 9 and 11. Consider an individual with $\hat{\theta}_{T-1} = \bar{\theta} - \epsilon$ and savings of at least ϵ, for some $\epsilon > 0$. The value of continuing is, using the proof of Proposition 10, $V_{T-1}(\bar{\theta} - \epsilon)$. This is strictly positive for small enough ϵ since, by Proposition 9, $\psi_{T-1} < \psi_T$. And since $V_t(\bar{\theta} - \epsilon) > V_{t+1}(\bar{\theta} - \epsilon)$ for any $\epsilon > 0$, then any individual with $\hat{\theta}_t = \bar{\theta} - \epsilon$, with $t < T-1$, and with savings of at least ϵ will also not exit.□

The exit policy that is induced by $w\left(\bar{\theta}\right) = w_0$ is only *privately* optimal. Anyone with sufficiently large savings will follow the privately optimal exit policy. From now on, denote this privately optimal exit policy of the credit constrained case by $\boldsymbol{\psi}^* \equiv \{\psi_1^*, \ldots, \psi_T^*\}$, where we know that $\psi_1^* < \cdots < \psi_T^* = \bar{\theta}$. However, some individuals—including all whose estimated talent is below the population mean after first period of work—are not able to do so because of lack of savings.

I assume that even risk neutral individuals need to consume at least w_0 every period, but that they don't mind saving the excess until retirement. (This extreme assumption about the saving rate is made to keep the notation simple, the same qualitative results are obtained as long as a positive fraction of rents is saved.) Savings are useful by making it possible to follow the individually optimal exit policy in the future, should the individual's talent dip below the population mean but not so much as to go below ψ_t^*. These previously successful veterans, or "has-beens," are able to compete against novices for scarce job slots, who would pay more for the job if only they had the money. However, just having enough funds to pay for the next period's job is, in general, not enough for continuation to be worthwhile. This is because the expected benefits of continuation come in part from possible future paths where the individual gets positive rents only several periods from now.

Denote the savings of an individual with a history $\hat{\boldsymbol{\theta}}_{t-1} \equiv \{\hat{\theta}_1, \ldots, \hat{\theta}_{t-1}\}$ by $S_t(\hat{\boldsymbol{\theta}}_{t-1})$. Since individuals save all of the rents we have $S_t(\hat{\boldsymbol{\theta}}_{t-1}) = \sum_{s=1}^{t-1}\left(\hat{\theta}_t - \bar{\theta}\right)$. Denote the necessary and sufficient amount of savings for an individual with $\hat{\theta}_t$ to choose to continue by $W_t\left(\hat{\theta}_t\right)$. This is only defined for $\hat{\theta}_t \geq \psi_t^*$, because, below the threshold, the individual will want to exit regardless of savings.

Proposition 13 *The minimum wealth requirement W_t for an individual with expected talent $\hat{\theta}_t \geq \psi_t^*$ to choose to continue at time t satisfies the following properties.*

$$\begin{array}{ll} \textit{(i)} & W_t\left(\psi_t^*\right) = \sum_{s=t}^{T}\left(\bar{\theta} - \psi_s^*\right) \equiv W_t^* \\ \textit{(ii)} & \dfrac{\partial}{\partial\hat{\theta}} W_t\left(\hat{\theta}\right) < 0 \quad \textit{for } \hat{\theta} \in \left(\psi_t^*, \bar{\theta}\right) \\ \textit{(iii)} & W_t\left(\hat{\theta}\right) = 0 \quad \textit{for } \hat{\theta} \geq \bar{\theta} \end{array}$$

For proof of sufficiency of part (i), observe that W_t^* is the amount of spending under the worst case scenario, where expected talent evolves along the stopping policy. Thus an individual with savings W_t^* can never again be bound by the credit constraint—he will exit before retirement if and only if his talent falls below the stopping policy. For a proof of necessity, recall that an (unconstrained) individual of threshold talent gets zero expected rents by the definition of the exit threshold. Having savings that are less than W_t^* means that some of the possible future paths that are chosen by the unconstrained individual are not available; because these paths must have contributed positively to the expected rents, their removal will pull the expectation below zero. Part (ii) follows already from the fact that the current period payment for continuation is lower for a higher talent, and future prospects are at least as good. Part (iii) is trivial, since above-mean types don't need to pay for jobs.

Definition 2. Has-beens. *Individuals whose expected talent is below the population mean, but who are willing and able to pay for a job:* $\hat{\theta}_t \in [\psi_t^*, \bar{\theta})$ and $S_t(\hat{\boldsymbol{\theta}}_{t-1}) \geq W_t(\hat{\theta}_t)$.

A has-been is currently below the population mean but above the privately optimal exit policy. He must have once been successful enough to have sufficient funds to continue. In Figure 2, the potential has-beens are in the area between the horizontal line and the privately optimal stopping policy $\boldsymbol{\psi}^*$. The solid line shows a career path for a has-been that decides to continue at tenure t, and two possible continuations after that (one leading to early exit and the other to resurrection). The dashed line shows a career path for someone who is more talented (by expectation) at tenure t but nevertheless exits. He has all the same and even better chances at resurrection in principle, but his lower savings mean that he would have a higher risk of running out of funds before resurrection.[34]

[Figure 2: Mediocrities and Has-beens]

The dependence of the exit decision on previous success and wealth implies a rather peculiar correlation of features in the workforce. Among those below the population mean by talent, there is a negative correlation between talent and past success. This results from selection by wealth: it takes deeper pockets, and so more past success, to be able to continue for a lower level of below-mean talent. This selection can also be expressed in terms of the time profile of individual output history. Consider two individuals with the same expected talent $\hat{\theta}_t$ and same cumulative output $\sum_{s=1}^{t-1} y_s$, but with one having been an early success and the other a late bloomer. The one with better recent performance is the one with less wealth and is thus more prone to exit at a given $\hat{\theta}_t$. The reason is that good luck early on has a higher impact on lifetime wealth, first because the posterior reacts more to a single observation early on when there are fewer observations (so the variance of the prior is higher), and second because any performance will affect the wages of all subsequent periods (i.e. early luck gets counted into the wage more times over the career).[35]

[34] The dotted area is W_t^*, the future spending under the worst-case scenario for a threshold type has-been at time-t.

[35] This path-dependence intuition is a perverse counterpart to the later-beginner effect in Chiappori, Salanie and Valentin (1999). There wages are downward rigid and, of two workers with the same wage, the worker with better recent performance has better future prospects.

The presence of any has-beens in the workforce means that the efficiency loss in terms of average talent in the industry is greater than if saving was not possible. Just like mediocrities, has-beens reduce total output in the long run (by having less upside potential than novices), but unlike mediocrities they also reduce it in the near term (by being worse than novices by expectation).

Where could we observe "has-beens" in the sense defined here?[36] Serial entrepreneurs who use profits from earlier ventures to finance a new start-up after recent failures can be has-beens. In the movie industry, a has-been is an actor or a director who used to be a star and made large talent rents, but has flopped more recently. He then uses savings from earlier rents to participate in the financing of a movie, which makes negative profits in expectation, but in return offers him a role and a chance at a resurrection.[37] For him this gamble has a positive expected value, because a successful comeback would generate more talent rents in the future.

Interpretation as One-Sided Long-Term Contracts Suppose firms commit to a lifetime wage-policy, including a severance payment policy, even though individuals cannot commit to contracts that require them to make payments to the firm at any time in the future (e.g., no quitting penalty). Now the accumulated wealth S_t would be analogous to money "in escrow" at the firm, which must always be nonnegative. In the simplest contract individuals would get w_0 until they exit, upon which the firm pays out S_t. Some separations result from insufficient funds in escrow, but even those typically involve severance payments by the firm. When the escrow is full (i.e., $S_t \geq W_t^*$) exit is voluntary: the worker quits to stop the bleeding of the escrow because he has fallen below the privately optimal exit policy ψ_t^*.

More interestingly, the contract could also include wages above w_0 before separation or retirement. For sufficiently good histories, the escrow balance can reach a point where no amount of bad news in the future could ever cause the individual to be fired due to insufficient funds. In terms of the spot contract world, the credit constraint can no longer become binding because $S_t \geq W_t^*$, even though the privately optimal exit policy can still become binding after sufficiently bad performance. This allows the firm to start unloading the account with payments above w_0, up to the point where the remaining balance is W_t^*.

The result that a worker's escrow can reach a firing-proof level W_t^* is reminiscent of the "tenure standard" of Harris and Weiss (1984), but this is a different phenomenon. In their paper, for a sufficiently good history of performance, the expected marginal product

[36] In MacDonald and Weisbach (2004), the term "has-beens" is used for individuals with outdated vintage human capital.

[37] David Autor suggested calling this "the Kevin Costner effect," but the fairness of this term is an open empirical question and thus remains in the footnote.

of a worker reaches a level at which the firm knows it can never again fall below the outside wage. A crucial assumption there is that output consists of successes that arrive as a Poisson process; the impact of the worst possible news is therefore bounded below.

Firms' ability to commit to long-term contracts does not improve efficiency here, it merely allows a different interpretation of the equilibrium. In a setup with credit-constrained individuals and unobservable effort, an escrow can serve the useful purpose of (imperfectly) mimicking an up-front performance bond, as proposed by Akerlof and Katz (1989). Also, if individuals were both credit-constrained and risk averse, then one-sided long-term contracts would allow firms to provide insurance, as in Harris and Holmström (1982). The difference here is that wage insurance against type realizations below ψ_T is provided by the outside wage (turning out to be a bad actor does not diminish one's prospects as a dishwasher). Note that there is no scope for wage insurance in the basic model, because workers do not face downside risk: novices know that they will make at least their current wage in the future, and veterans know that their type (and wages) will stay constant until retirement.

Conclusion From the point of view of efficiency of talent discovery, the only substantive difference between one-shot learning and gradual learning is the possibility to accumulate savings under the latter. Surprisingly, the opportunity to save aggravates the inefficiency caused by a credit constraint. Saving by "has-been" individuals who perform well early in their career, but who fall below population mean in expected talent later, allows them to outbid credit constrained novices. Their incentive to pay for jobs is the chance of more talent rents in the future: as talent is only revealed over time, the has-beens still retain some upside potential, albeit less than the novices. However, after sufficiently bad performance even the has-beens exit, regardless of their savings.

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Gradual learning. Optimal exit policy.

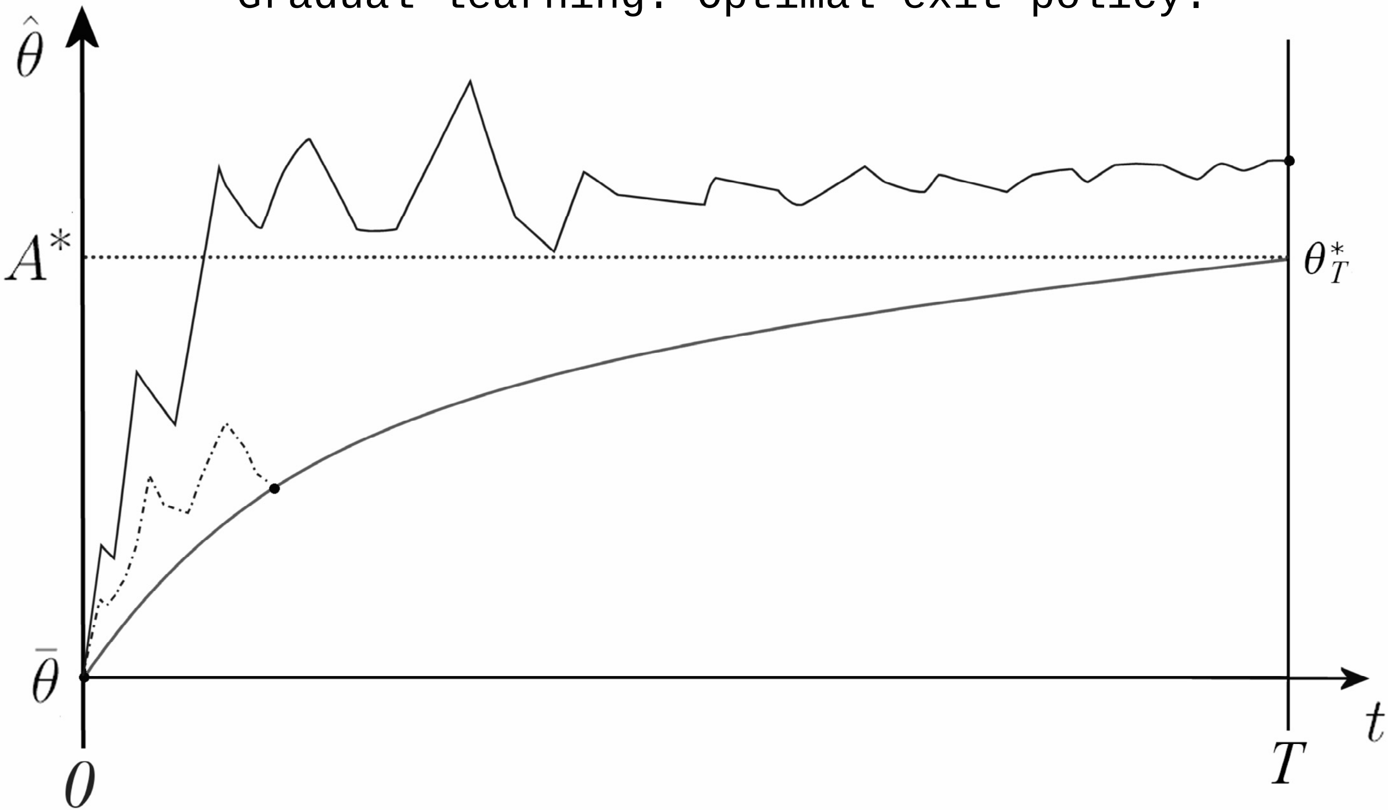

Figure 1. *Efficient Benchmark.*

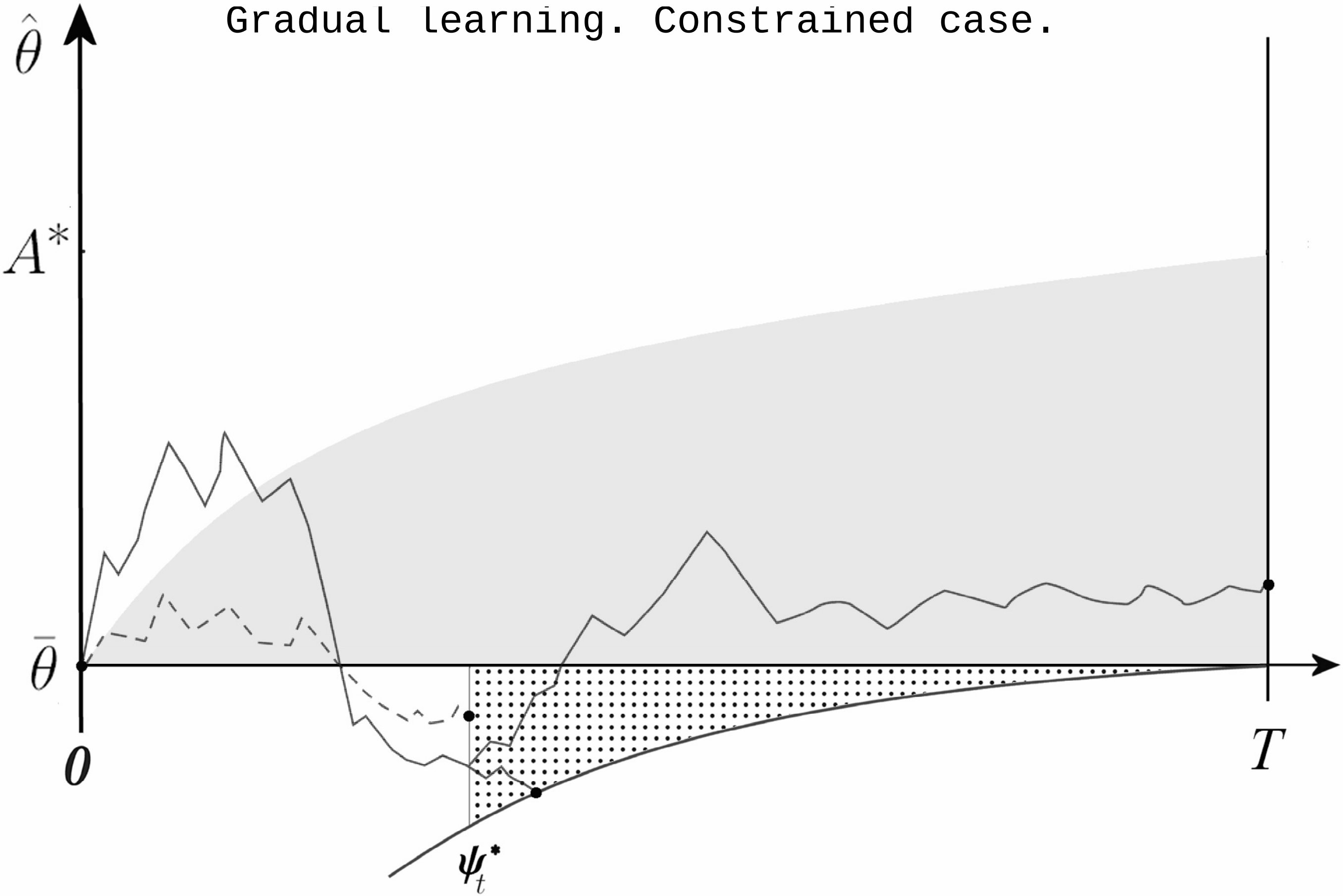

Figure 2. *Mediocrities and Has-beens.*