31E00700 Labor Economics: **Lecture 7**

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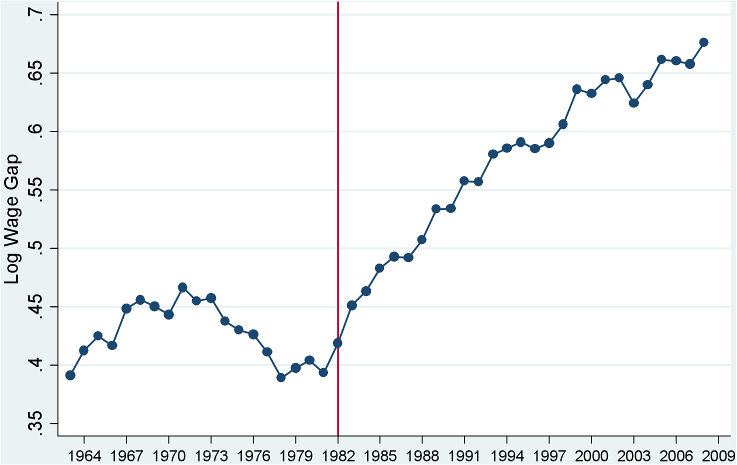
First Part of the Course: Outline

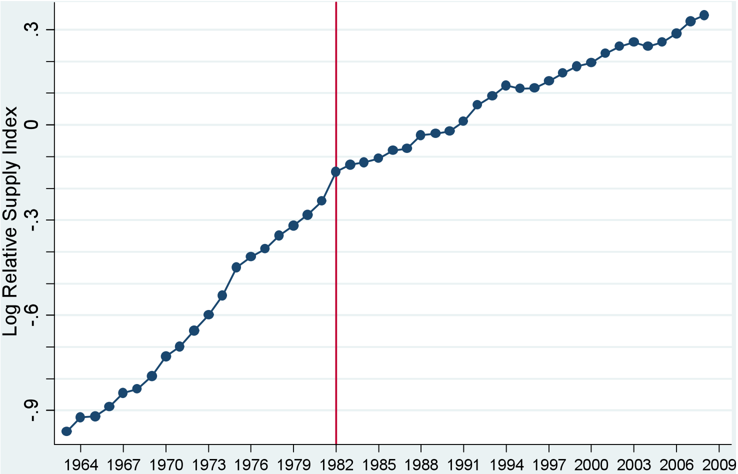
1. Supply of labor
2. Demand for labor
3. Labor market equilibrium
    1. Perfectly competitive markets; immigration
    2. Imperfectly competitive labor markets; minimum wages
    3. **Technological change and polarization (today)**

Stylized Fact 1: Returns to skill have increased



Composition adjusted college/high-school log weakly wage ratio (holds constant the relative employment shares by gender, education and potential experience). Source: Acemoglu and Autor (2011): Skills, Tasks and Technologies: Implications for Employment and Earnings. Handbook of Labor Economics

Stylized Fact 2: Supply of skills have increased



College/High-school log relative supply. Source: Acemoglu and Autor (2011): Skills, Tasks and Technologies: Implications for Employment and Earnings. Handbook of Labor Economics.

The "Canonial Model" of SBTC

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  - Labor divided to skill groups (typically just high/low skilled)
  - **SBTC** (skill-biased technological change): new technologies increase the productivity of high-skilled workers more
  - Wages determined in competitive labor markets

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  - **SBTC** (skill-biased technological change): new technologies increase the productivity of high-skilled workers more
  - Wages determined in competitive labor markets
- Tinbergen's "race between technology and education"
  - SBTC increases high/low-skilled wage gap
  - Increasing education decreases the gap (by increasing the relative supply of high-skilled workers)

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- L, H are aggregate efficiency units of low, high skilled labor
- A_l, A_h are *factor-augmenting* technology terms
- $\rho = \frac{\sigma - 1}{\sigma}$ is a convenient way to put elasticity of substitution between high skill and low skill labor, $\sigma \in [0, \infty)$
  - Low, high skilled workers gross substitutes if $\sigma > 1$ (i.e. $\rho > 0$)
  - ... and gross complements if $\sigma < 1$ (i.e. $\rho < 0$)

More on CES production function

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  - $\sigma \to 1$ ($\rho \to 0$): Cobb Douglas

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- Competitive labor markets $\rightarrow$ wages = marginal product

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  - $\frac{\delta w_H}{\delta A_h} > 0$ and $\frac{\delta w_H}{\delta A_l} > 0$. **Either kind of factor-augmenting technical change increases wages of both skill types** [Increase in own productivity increases own wages. Increase in the productivity of the other skill type is identical to an increase in the number of workers of the other skill type, see above]

Skill Premium: Changes in Supply of Skills

- Using the wage equations above

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- Note that $\log\left(\frac{w_H}{w_L}\right) = \rho \log\left(\frac{A_h}{A_l}\right) + (\rho-1)\log\left(\frac{H}{L}\right)$, so we get a simple form for how relative supplies affect relative wages:

$$\frac{\delta \log(w_H/w_L)}{\delta \log(H/L)} = (\rho-1) = -\frac{1}{\sigma}$$

where σ is the elasticity of substitution between high skill and low skill labor. Since $\sigma \geq 0$, the ***relative* demand curve is downward sloping**, i.e. for a given technology, an increase in relative supplies lowers relative wages.

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- How could an increase in A_h decrease skill premium?
  - If $\rho < 0$, high and low skilled workers are gross complements
  - An extreme case: when $\rho \to -\infty$, CES-production function approaches Leontief. Thus an increase in $A_h H$ would create "excess supply" of high skilled workers for a given number of low-skilled workers.

- However, there seems to be a consensus that $\rho > 0$, in which case an increase in A_h/A_l increases skill premium

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  - increase wages of unskilled workers
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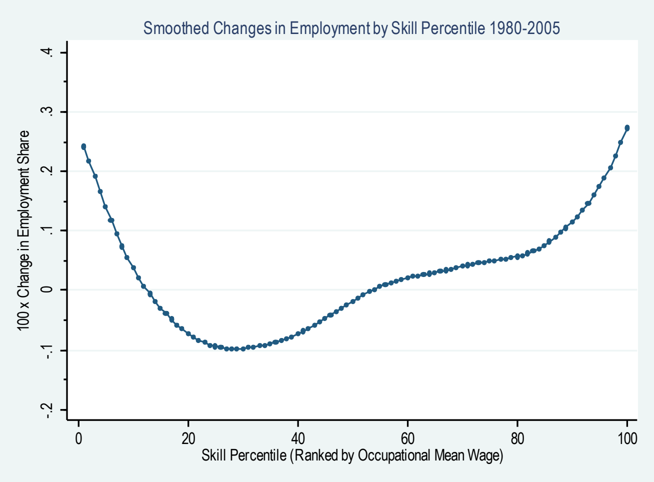
- Results generalize to adding more factors (e.g. capital)
- Katz and Murphy (1992), Card and Lemieux (2001) probably the most famous papers taking this model to data

Stylized Fact 3: Polarization, employment, US



Source: Autor and Dorn (forthcoming)

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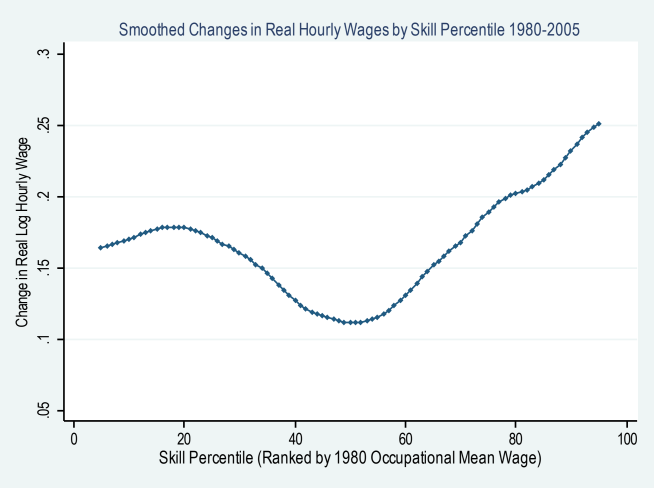

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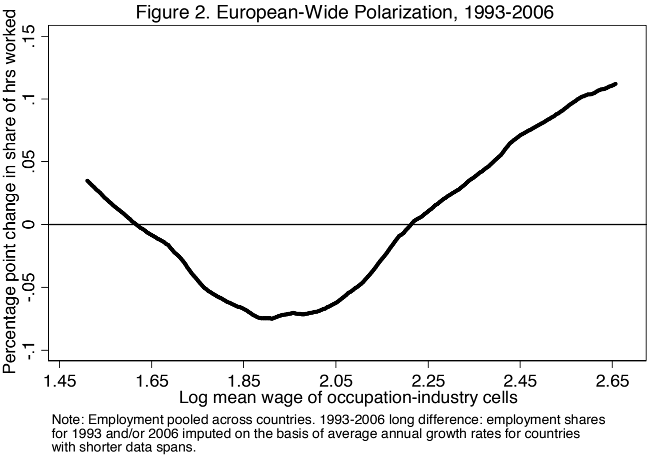
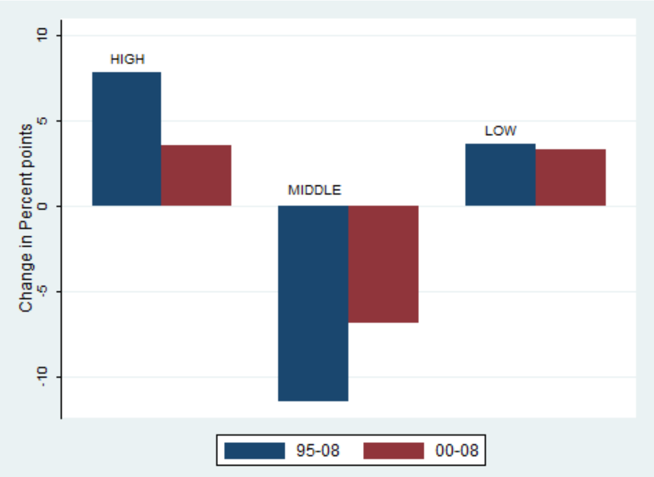
Polarization, employment, Europe



Source: Goose, Manning, Salomons (2010)

Polarization, employment, Finland



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Can SBTC explain polarization?

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- Automation technologies may replace **tasks** previously performed by workers in the middle of the income distribution $\rightarrow$ workers pushed from middle to low/high-income jobs
- To model this possibility, one needs to go beyond the canonial model
- Recently, a lot of papers using tasks to examine skill demand, technological change, offshoring, international trade
  - Distinction between what workers know (skills) and what they actually do (tasks)
  - Self-selection into tasks based on comparative advantage
  - Autor, Levy and Murnane (2003), Costinot and Vogel (2009), Acemoglu and Autor (2011) and many others
- We next discuss Autor and Dorn (forthcoming) in some detail.
  [You are not expected to be able to derive the formal results, but need to get the intuition of the model and the basic empirical results]

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- **Goods produced with abstract and routine tasks**

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where L_a is the total amount of abstact tasks, L_r total amount of routine task, K capital and αs productivity parameters

  - Cobb-Douglas, where routine tasks performed by workers and computers are aggregated with CES-function (i.e. μ depends on the elasticity of substitution /bw computers and workers)

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- **Services produced with manual tasks**

$$Y_s = \alpha_s L_m$$

Selection

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- Self-selection based on comparative avantage
  - low skilled worker provides routine tasks iff $\eta_i w_r \geq w_m$
  - everyone above the threshold $\underline{\eta} \geq \frac{w_m}{w_r}$ work in the goods sector
  - wages in goods sector higher than in services

Technological progress and Consumer utility

- At period t, the amount of computer capital is

$$K = Y_k e^{\delta t}/\theta$$

  where Y_k is the amount of final consumption good allocated to production of K, $\delta > 0$ is a constant, t is the calendar year and θ is a productivity parameter

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- Note that productivity in capital production, $\delta t/\theta$, is increasing over time $\rightarrow$ the price of computers falls (eventually to zero)
- To close the model, we also need to specify workers (who are also consumers) utility function. Assume CES-form

$$u = (c_s^\rho + c_r^\rho)^{1/\rho}$$

where $\rho = \frac{\sigma - 1}{\sigma} < 1$ is determined by elasticity of substitution in consumption

Equilibrium

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- Optimization and FOCs
  - Since there are no frictions, equilibrium allocation can be characterized by solving the social planners problem
  - This simplifies the problem substantially, but the first order conditions are still quite messy (see the paper)
  - Asymptotical (time goes to infinity) results much sharper

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- It follows that asymptotic allocation of low of skilled labor to services is

$$L_m = \begin{cases} 1 & \text{if } \frac{1}{\rho} > \frac{\beta - \mu}{\beta} \\ \overline{L}_m \in (0,1) & \text{if } \frac{1}{\rho} = \frac{\beta - \mu}{\beta} \\ 0 & \text{if } \frac{1}{\rho} < \frac{\beta - \mu}{\beta} \end{cases}$$

where $\rho = \frac{\sigma - 1}{\sigma}$ depends on elasticity of substitution between goods and services in *consumption*, $\mu = \frac{\sigma_r - 1}{\sigma_r}$ depends on the elasticity of substitution between capital and labor in the production of routine tasks aggregate, and β is the share of the routine aggregate in goods production

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- Simplest case: $\beta = 1$. Everyone will work in services iff $\sigma_r > \sigma$, i.e. if elasticity of substitution between labor and capital in production is higher than elasticity of substitution between goods and services in consumption.

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$$\frac{w_a}{w_m} = \begin{cases} 0 & \text{if } \sigma < 1 \\ 1 & \text{if } \sigma = 1 \text{ and } \frac{1}{\sigma} > \frac{\beta-\mu}{\beta} \\ \infty & \text{if } \sigma > 1 \end{cases}$$

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- If goods and services are gross complements, $\sigma < 1$, low-skilled service workers will eventually earn infinitely more than high-skilled workers [there will be an infinite amount of goods, but a limited amount of services and abstract tasks are used only in the goods sector]
- Despite the marginal *physical* product of high skilled workers going to infinity, their marginal *value* product does not

Empirical implications

- The model above too stylized to be taken directly to data
- An extension considers several integrated markets in a spatial equilibrium setting
    - each location produces a differentiated good
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    - high skill workers mobile, low skilled immobile

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- Testable predictions: Local labor markets starting with greater specialization in routine tasks should
    - adopt information technology faster
    - reallocate more low skilled workers to services
    - experience more wage polarization
    - experinece more high-skilled inmigration

Empirical results

- Commuting zones (CZ) that had high share of low-skilled workers in routine tasks in 1980
  - adopted computers faster in 1980–2005 (Table 3, panel A)
  - experienced faster decline in routine occupations, particularly among workers without college education (Table 3, panel B)
  - experienced faster growth in service occupations (Figure 6, Table 4)
    - note that in 1950–1970, this association was the opposite (Table 4)
    - and that instrumenting for the initial routine task intensity of CZ makes the results stronger
  - ... and occupations with low routine content (Table 7, panel A)
  - experienced faster wage growth in occupations with low routine content (Table 7, panel B)
- Alternative explanations (Table 6)
  - "Offshorability" of occupations does not explain the growth of services across CZS
  - proxies for income and substitution effects do not have a strong direct relationship with growth in services

Appendix: Deriving the marginal product of labor from a two factor CES production function

Taking the partial derivate of $Y = ([A_l L]^\rho + [A_h H]^\rho)^{1/\rho}$ with respect to L yields (just use the chain rule)

$$\begin{aligned} \frac{\delta Y}{\delta L} &= \frac{1}{\rho}([A_l L]^\rho + [A_h H]^\rho)^{\frac{1}{\rho}-1} \times \rho [A_l L]^{\rho-1} \times A_l \\ &= \left(A_l^\rho L^\rho + A_h^\rho H^\rho\right)^{\frac{1-\rho}{\rho}} \times L^{\rho-1} {A_l}^\rho \\ &= A_l^\rho \left[A_l^\rho + A_h^\rho (H/L)^\rho\right]^{\frac{1-\rho}{\rho}} \end{aligned}$$

where the last step uses the identity $L^{\rho-1} = (L^{-\rho})^{(1-\rho)/\rho}$